International Journal of Theoretical Physics, Vol. 31, No. 8, 1992

Large Number Hypothesis

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Received February 15, 1991

Dirac's large number hypothesis (LNH) and the Whitrow-Randall-Sciama relation are related to some cosmological models. The LNH and Whitrow-Randall-Sciama relations are neither equivalent nor consistent in general relativity, but they may both be valid in the Brans-Dicke theory and in another theory considered in this paper.

The large number hypothesis (Dirac, 1938, 1979) states that

$$\rho \propto t^{-1} \tag{1}$$

$$G \propto t^{-1} \tag{2}$$

where ρ, G, and t stand for the rest-energy density, the gravitational "constant," and the cosmic time.

It is useful to recall the origin of these relations. One can build the following dimensionless numbers:

$$\frac{GH_0^{-1}}{e^2/m_e c^2} \approx 10^{40} \tag{3}$$

$$\frac{e^2}{Gm_p m_e} \approx 10^{40} \tag{4}$$

$$\frac{\rho_0 (GH_0^{-1})^3}{m_p} \approx 10^{2\times 40} \tag{5}$$

where e, m_e, m_p, and ρ_0 stand for the experimentally measured values of the electronic charge and mass, the protonic mass, and the present-day ρ.

On the other hand, H_0 stands for the present-day Hubble "constant." If (3)–(5) are not accidental coincidences, then (1) and (2) are true relations.

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0020-7748/92/0800-1447$06.50/0

Whitrow (1946), and Whitrow and Randall (1951) advanced simple arguments that show that we should have, for the present universe, the relation

$$G\rho \sim t^{-2} \tag{6}$$

and Sciama (1953) represented Mach's principle by the same relation (6). Later, Dicke (1962), arguing on simple dimensional arguments plus Mach's principle, arrived at relation (6) again.

Teller (1948) made an estimate of the thermal history of the earth which, with the data available at the time, seemed to indicate that relation (2) did not hold. Later data on Hubble's "constant," however, could support relation (2) (Raychaudhuri, 1979). A very neat and up-to-date account on the LNH is to be found in Barrow (1990).

It should be clear that relations (1) and (2) imply (6), but is it possible to have (6) and not have (1) and (2)?

The answer to this question, in Einstein's general relativity theory, is yes—of course.

When we write

$$p = \alpha\rho \qquad (\alpha = \text{const}) \tag{7}$$

where p stands for the cosmic pressure, we find that, if

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = m - 1 = \text{const} \tag{8}$$

a solution is given by (Berman, 1983; Berman and Gomide, 1988)

$$R = (mDt)^{1/m} \qquad (D = \text{const}) \tag{9}$$

$$\rho = \frac{3}{8\pi G m^2}\, t^{-2} \tag{10}$$

$$k = 0 \qquad (\text{null tricurvature})$$

In the Brans–Dicke theory, the Whitrow–Randall relation, plus the assumption $q = \text{const}$, leads to relations (1) and (2) (Berman and Som, 1990).

In fact, Berman and Som (1990) show that a solution can be found where relation (7) is also satisfied if in relation (6) the "approximate" sign is substituted by an equality. For this particular case, the coupling constant w of the Brans–Dicke theory can only have the positive value $w \cong 1.12$ or $w \cong 1.69$.

However, if we do not restrict relation (6) to be an equality relation, larger values of w are also possible, and the experimentally obtained bound $w > 500$ is possible in this framework. It has also to be pointed out that

Berman and Som (1990) offer not only flat models, but also nonflat ones, as possible solutions obeying relations (1) and (2).

Now, what about Λ and G variables in an Einstein-like theory?

Berman (1991) has shown that, when one supposes that the energy tensor conservation law is valid, as well as the Einstein field equations,

$$G_{\mu\nu} = 8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu} \tag{11}$$

where $G_{\mu\nu}$, $g_{\mu\nu}$, $T_{\mu\nu}$, and Λ are respectively, the Einstein, metric, and energy tensors and the cosmological "constant," and we suppose a perfect fluid law

$$T_{\mu\nu} = -pg_{\mu\nu} + (p+\rho)U_\mu U_\nu \tag{12}$$

we can find solutions where relations (1) and (2) hold. We then have

$$R(t) = (mDt)^{1/m} \tag{13}$$

$$p = \alpha\rho \tag{14}$$

$$k = 0 \tag{15}$$

$$G = Kt^{B/4\pi A} \tag{16}$$

$$\rho = \frac{A}{K}t^{-(2+B/4\pi A)} \tag{17}$$

$$m = \left(\frac{3}{8\pi A + B}\right)^{1/2} \tag{18}$$

$$\Lambda = Bt^{-2} \tag{19}$$

$$\alpha = \frac{1}{3}\left[m\left(2+\frac{B}{4\pi A}\right)^{-1}\right] \tag{20}$$

where K, A, and B are constants.

In order to fulfill relations (1) and (2), we need

$$\frac{B}{4\pi A} = -1 \tag{21}$$

This implies, from relation (20),

$$\alpha = \tfrac{1}{3}(m-1) \tag{22}$$

Relation (22) ensures that for each phase of the standard cosmological model we have a different power law for the scale factor R. In fact, the present universe, for which we may state that $\alpha = 0$, implies $m = 1$. This is a "nonaccelerated" universe ($q = 0$). Remember that in standard cosmology, we have a different result, *verbi gratia*, $m = 3/2$. For the radiation phase, $\alpha = 1/3$, and then we would have $m = 2$. This result is the same as in standard cosmology.

Let us now check the particle-creation rate of our present universe for this model:

$$r = \frac{d}{R^3\,dt}(\rho R^3) = 2\rho H \tag{23}$$

where $H = 1/t$ is the Hubble parameter (recall that $m = 1$).

Our result can be compared with steady-state cosmology, where we have a rate $3/2$ larger,

$$r_{ss} = 3\rho H \tag{24}$$

Narlikar (1983) states that the order of magnitude of r_{ss} (and, thus, of r), is 10^{-41} g cm^3 sec^{-1}, which is completely inaccessible to experimental verification, and thus the present theory cannot be ruled out on experimental grounds for the time being.

ACKNOWLEDGMENT

This work was partially funded by CNPq (Brazilian government agency).

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